1. Suppose that you have the following ARMA(1,2) model

$$x_t = 0.36 + 0.8x_{t-1} + u_t - 1.1u_{t-1} + 0.24u_{t-2} \tag{1}$$

where $u_t \sim \text{WN}(0, 1)$. **[10 marks]**

   (a) Write the model in lag operator form and find the unconditional mean of the process. **[2 marks]**

   (b) Find the variance ($\gamma_x(0)$) and the 1^{st} to 3^{rd} autocovariances ($\gamma_x(1), \gamma_x(2)$ and $\gamma_x(3)$) of the process. **[5 marks]**

   (c) Find the 1 to 3 step ahead forecasts from the model in (1). You can assume for simplicity that all observed u_t and x_t are equal to 1 when you make the forecast. Find also the 1 to 3 step ahead forecast errors as well as the forecast error variances. **[3 marks]**

2. Suppose you have the following *structural time series model* for x_t

$$x_t = \mu_t + \varepsilon_t \tag{2a}$$
$$\mu_t = \alpha\mu_{t-1} + \eta_t + v_t \tag{2b}$$
$$\eta_t = \beta\eta_{t-1} + u_t \tag{2c}$$

where the three shocks u_t, v_t, and ε_t are each $\text{WN}(0, 1)$ and are also mutually uncorrelated at all time horizons. **[5 marks]**

   (a) What type of ARMA(p, q) model do the relations in (2) imply for x_t? Use time series algebra to prove your answer. **[5 marks]**

3. Suppose you have the following *quadratic trend model* for x_t

$$x_t = c + \alpha x_{t-1} + \delta_1 t + \delta_2 t^2 + u_t \tag{3}$$

where $u_t \sim \text{WN}(0, 1)$ and $|\alpha| < 1$. **[6 marks]**

   (a) How many times do you need to difference the series so that the trend component is removed? Use time series algebra to prove your results. **[3 marks]**

   (b) After you have differenced the series, what other problems do you think may arise when trying to model the differenced data series. **[3 marks]**

4. Suppose you want to model the relationship between x_{1t} and x_{2t} within a system that takes the form

$$x_{1t} = a_{12}^0 x_{2t} + a_{11}^1 x_{1t-1} + a_{12}^1 x_{2t-1} + \varepsilon_{1t} \tag{4}$$
$$x_{2t} = a_{21}^0 x_{1t} + a_{21}^1 x_{1t-1} + a_{22}^1 x_{2t-1} + \varepsilon_{2t} \tag{5}$$

and you make the assumption that shocks to the first equation have only a transitory effect on x_{2t} while those of the second equation have a permanent effect. **[8 marks]**

(a) Follow the Blanchard-Quah approach and use the transitory effect of ε_{1t} on x_{2t} as a restriction on the coefficients of the VMA(∞) representation to identify the model. What do these restrictions look like when you write them down? **[4 marks]**

(b) Describe exactly how you would use the restrictions found in (a) to estimate the parameters of the model above and tell me what assumptions you made. **[4 marks]**

5. Consider the following MA(∞) model

$$x_t = u_t + bu_{t-1} + b\rho u_{t-2} + b\rho^2 u_{t-3} + b\rho^3 u_{t-4} + \dots \tag{6}$$

where $u_t \sim \text{WN}(0, 1)$ and $|\rho| < 1$. **[6 marks]**

(a) For which value of b is the MA(∞) in (6) an AR(1). **[3 marks]**

(b) Show that for any other values of b found in (a) the process is an ARMA(1, 1) and identify the MA and AR parameters. **[3 marks]**

6. Suppose you have three variables that form a cointegrating relation of the form:

$$x_{1t} = \beta_2 x_{2t} + \beta_3 x_{3t} + u_{1t} \tag{7}$$

$$x_{2t} = \sum_{i=1}^{t} u_{2i} \tag{8}$$

$$x_{3t} = x_{3,t-1} + u_{3t} \tag{9}$$

where the $u_{jt}, \forall j = 1, 2, 3$ are white noise disturbance terms. **[5 marks]**

(a) Find the cointegrating vector(s) above and verify that a stationary linear combination of the x_{jt} components is obtained, $\forall j = 1, 2, 3$. Determine also the common stochastic trends that drive this system. **[5 marks]**