NKPM Model Notes

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1. Our Model

$$
\begin{aligned}
\pi_t &= \rho_\pi \mathbb{E}_t \pi_{t+1} + (1-\rho_\pi)\,\pi_{t-1} + \lambda_1 y_t + \lambda_2 q_t + \epsilon_{AS,t} \\
y_t &= \rho_y \mathbb{E}_t y_{t+1} + (1-\rho_y) y_{t-1} - \delta_1 \left(r_{t-1} - \mathbb{E}_{t-1}\pi_t\right) + \delta_2 q_{t-1} + \delta_3 y_t^* + \epsilon_{IS,t} \\
r_t &= \rho_r r_{t-1} + (1-\rho_r)\left(\psi_1 \mathbb{E}_t \pi_{t+1} + \psi_2 y_t\right) + \epsilon_{MP,t} \\
\mathbb{E}_t \Delta q_{t+1} &= \left(r_t - \mathbb{E}_t \pi_{t+1}\right) - \left(r_t^* - \mathbb{E}_t \pi_{t+1}^*\right) + \epsilon_{RER,t}
\end{aligned}
$$

$$
\begin{aligned}
\pi_t^* &= \rho_\pi^* \mathbb{E}_t \pi_{t+1}^* + (1-\rho_\pi^*)\,\pi_{t-1}^* + \lambda^* y_t^* + \epsilon_{AS,t}^* \\
y_t^* &= \rho_y^* \mathbb{E}_t y_{t+1}^* + (1-\rho_y^*) y_{t-1}^* - \delta^* (r_{t-1}^* - \mathbb{E}_{t-1}\pi_t^*) + \epsilon_{IS,t}^* \\
r_t^* &= \rho_r^* r_{t-1}^* + (1-\rho_r^*)\left(\psi_1^* \mathbb{E}_t \pi_{t+1}^* + \psi_2^* y_t^*\right) + \epsilon_{MP,t}^*
\end{aligned}
$$

2. Svensson Model

The domestic block is

$$
\begin{aligned}
\pi_{t+1} &= \alpha_\pi \bar{\pi}_t + (1-\alpha_\pi)\,\mathbb{E}_t \pi_{t+2} + \alpha_x x_{t+1} + \alpha_q \Delta q_t + \epsilon_{t+1}^{cp} \\
x_{t+1} &= \beta_x x_t + (1-\beta_x)\,\mathbb{E}_t x_{t+2} - \beta_i \left(i_t - \mathbb{E}_t \pi_{t+1}\right) + \beta_{x^*} x_{t+1}^* + \beta_q \mathbb{E}_t q_{t+1} + \epsilon_{t+1}^{ad} \\
i_{t+1} &= \rho_i i_t + (1-\rho_i)\left[\gamma_x x_{t+1} + \gamma_\pi \bar{\pi}_{t+1} + \gamma_{i^*} i_{t+1}^* + \gamma_{x^*} x_{t+1}^* + \gamma_{\pi^*} \bar{\pi}_{t+1}^*\right] + \epsilon_{t+1}^{mp} \\
y_{t+1} &= \rho_f y_t^f + \rho_{f^*} y_t^{f^*} + \epsilon_{t+1}^f \\
q_t &= \mathbb{E}_t q_{t+1} - \left(i_t - \mathbb{E}_t \pi_{t+1}\right) + \left(i_t^* - \mathbb{E}_t \pi_{t+1}^*\right)
\end{aligned}
$$

where $x_t \equiv y_t - y_t^f$. The foreign or US block is

$$
\begin{aligned}
\pi_{t+1}^* &= \alpha_\pi^* \bar{\pi}_t^* + (1-\alpha_\pi^*)\,\mathbb{E}_t \pi_{t+2}^* + \alpha_x^* x_{t+1}^* + \epsilon_{t+1}^{cp^*} \\
x_{t+1}^* &= \beta_x^* x_t^* + (1-\beta_x^*)\,\mathbb{E}_t x_{t+2}^* - \beta_i^* \left(i_t^* - \mathbb{E}_t \pi_{t+1}^*\right) + \epsilon_{t+1}^{ad^*} \\
i_{t+1}^* &= \rho_i^* i_t^* + (1-\rho_i^*)\left[\gamma_x^* x_{t+1}^* + \gamma_\pi^* \bar{\pi}_{t+1}^*\right] + \epsilon_{t+1}^{mp^*}
\end{aligned}
$$

with $y_{t+1}^{f^*} = y_t^{f^*} + \epsilon_{t+1}^{f^*}$, $x_t^* \equiv y_t^* - y_t^{f^*}$, $\bar{\pi}_t = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4$, y_t log of real GDP, y_t^f log of real potential output at full employment.

3. Dungey and Pagan SVAR restrictions

3.1. Restrictions on the lag structure

Dungey and Pagan (2000) do not have foreign (U.S.) inflation π_t^* in their model, so that it is not clear what the restrictions are that would correspond to their model. I have put ? in the relevant fields below where there is no exact match to their restrictions. It is probably most plausible to leave those bits unrestricted for the U.S. block. I have marked those then with a ? $(*)$. Also, Dungey and Pagan (2000) use both GDP and GNE as income measures for Australia, with slightly different restrictions on them in the model. Note that Dungey and Pagan (2000) restrict the effect of GDP on $CASH$ and INF to be zero, while GNE is left unrestricted. I have decided to leave $y_{t-1,t-2}$ in the lag structure without restricting it to have zero effect on π_t and r_t, because we only have one income measure. This is also marked by ? $(*)$.

	$y^*_{t-1,t-2}$	$\pi^*_{t-1,t-2}$	$r^*_{t-1,t-2}$	$y_{t-1,t-2}$	$\pi_{t-1,t-2}$	$r_{t-1,t-2}$	$q_{t-1,t-2}$
y_t^*	$*$	?$(*)$	$*$	0	0	0	0
π_t^*	?$(*)$	?$(*)$	?$(*)$	0	0	0	0
r_t^*	$*$	?$(*)$	$*$	0	0	0	0
y_t	$*$	?	0	$*$	$*$	$*$	$*$
π_t	0	?	0	? $(*)$	$*$	0	$*$
r_t	0	?$(*)$	0	? $(*)$	$*$	$*$	$*$
q_t	$*$	?$(*)$	$*$	$*$	$*$	$*$	$*$

TABLE 1: These are the restrictions on the lag structure, not the contemporaneous restrictions.

3.2. Contemporaneous restrictions

Comments regarding ? $(*)$ are the same as above for the lag structure. Note that Dungey and Pagan (2000) use $RTWI$ as well as TOT as measures of international competitiveness in their SVAR, while we only have the real exchange rate q_t. Since q_t is a subset of $RTWI$, I have imposed the restrictions as for $RTWI$ in their paper by restricting its effect on domestic variables. This is marked by ? (0). For the U.S. block, a triangular ordering is used again as in the previous analysis.

	y_t^*	π_t^*	r_t^*	y_t	π_t	r_t	q_t
y_t^*	1	0	0	0	0	0	0
π_t^*	? $(*)$	1	0	0	0	0	0
r_t^*	$*$	? $(*)$	1	0	0	0	0
y_t	$*$	? (0)	0	1	0	0	? (0)
π_t	0	? $(*)$	0	? $(*)$	1	0	? (0)
r_t	? (0)	? (0)	? (0)	? $(*)$	$*$	1	? (0)
q_t	$*$	$*$	$*$	$*$	$*$	$*$	1

TABLE 2: These are the contemporaneous restrictions.

4. Variance decompositions

	$\epsilon_{IS,t}$	$\epsilon_{AS,t}$	$\epsilon_{MP,t}$	$\epsilon^*_{IS,t}$	$\epsilon^*_{AS,t}$	$\epsilon^*_{MP,t}$	$\epsilon_{RER,t}$
	NKPM						
y_t	72	8	1	16	0	0	3
π_t	2	51	0	20	0	0	27
r_t	27	20	11	24	0	0	18
	SVAR						
y_t	65	3	8	19	1	1	3
π_t	5	24	1	44	8	8	10
r_t	26	19	18	24	4	4	5
	Cholesky						
y_t	61	3	8	19	6	1	2
π_t	5	20	1	44	9	12	9
r_t	24	17	18	25	5	7	4

TABLE 3: Comparison of variance decompositions.

References

DE BROUWER, G. (1998): "Estimating Output Gaps." *Research Discussion Paper No. 98-09*, Reserve Bank of Australia.

DUNGEY, M. H. AND A. R. PAGAN (2000): "A Structural VAR Model of the Australian Economy." *The Economic Record*, 76(235), 321–342.

HARDING, D. AND A. R. PAGAN (2002): "Dissecting the Cycle: A Methodological Investigation." *Journal of Monetary Economics*, 49(2), 365–381.

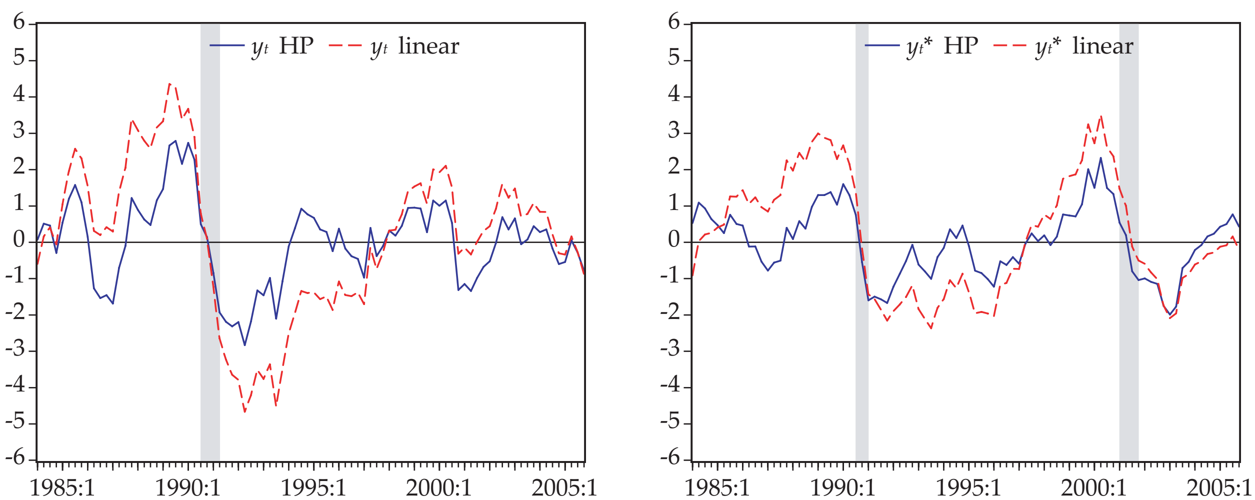
5. Lag length selection of VAR(2) model

Lag	LR	FPE	AIC	SBC	HQC
1	NA	0.003639	14.24607	15.68424*	14.82347*
2	89.17255*	0.003307*	14.12983*	17.00616	15.28463
3	59.50921	0.004365	14.34940	18.66388	16.08160
4	60.22034	0.005322	14.42933	20.18197	16.73893
5	60.06184	0.006036	14.34654	21.53734	17.23354
6	46.13104	0.008847	14.38838	23.01735	17.85278

TABLE 4: Lag length selection

6. Construction of output gaps

The construction of output gaps is still contentional, without any obvious solution provided in the literature (see de Brouwer, 1998, for a discussion relevant to Australian data). To give an indication of how different these can be, we show a plot of HP-filtered and linearly detrended output gaps for Australia and the U.S. in Figure 1 over the period 1984:Q1 to 2005:Q4. The shaded regions indicate recession dates, which were computed according



(a) Australia

(b) U.S.

FIGURE 1: Comparison of Australian and U.S. output gaps 1984:Q1 – 2005:Q4.

to the business cycle dating procedure of Harding and Pagan (2002) for Australia, while for the U.S., official NBER recession dates were used. Although the two series track one another quite closely, with correlation coefficients being respectively 0.80 and 0.82 for Australia and the U.S., the linearly detrended output gap has a higher variance and there are also likely differences in the interpretation of the state of the economy being below or above potential output.

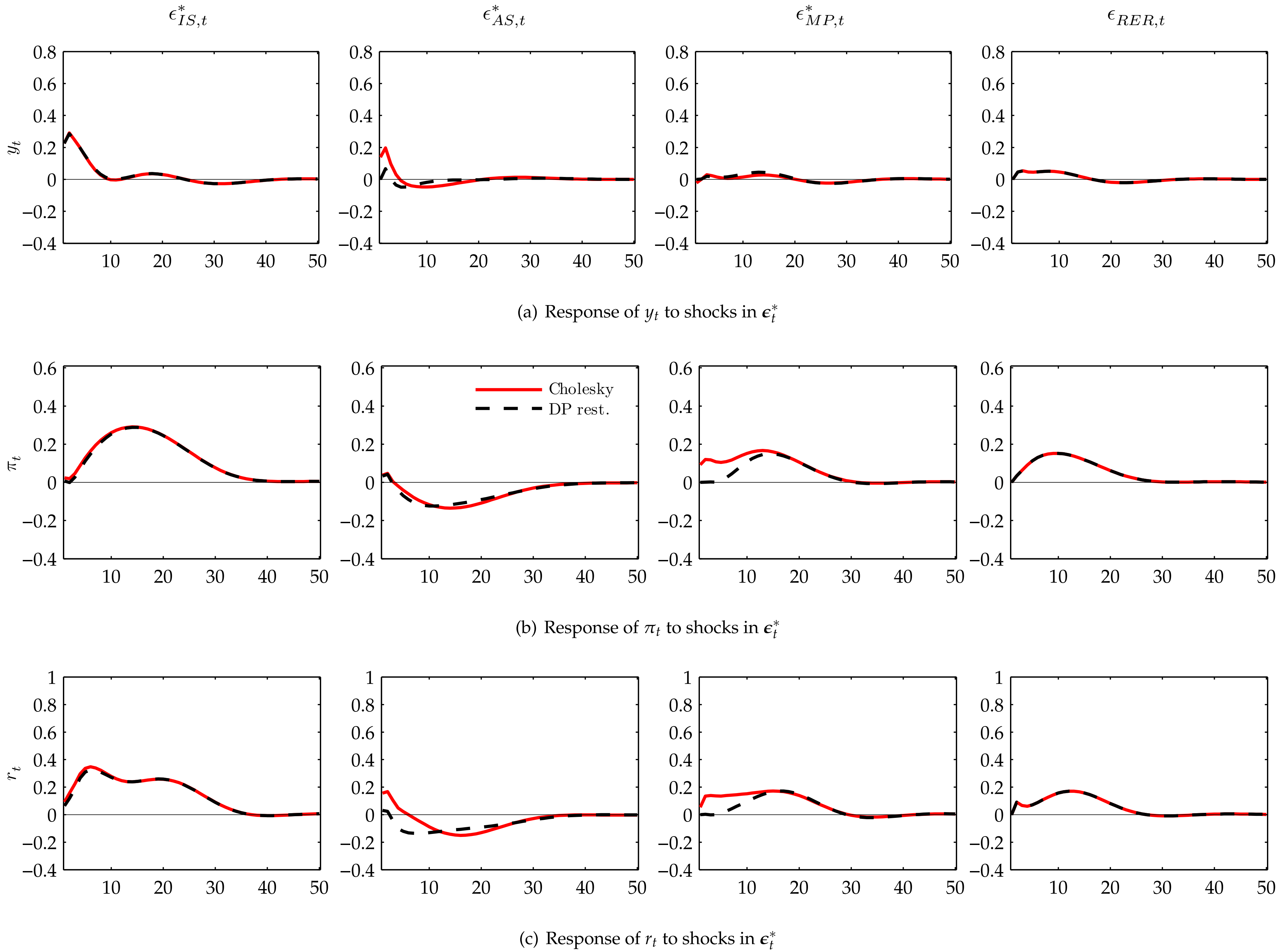
7. Comparison of DP and Cholesky restriction IRFs, and MLE and Bayesian NKPM ones

Lag restrictions are imposed as per Dungey and Pagan (2000), so we are only comparing the effect of the contemporaneous restrictions. Also, U.S. block has recursive identification structure.

Parameter comparison of MLE and Bayesian NKPM is given below.

(1)		(2)		(3)
Parameter		Bayesian Posterior		MLE
	Prior	Mean	95% CI	
ρ_π	$\mathcal{B}(0.50, 0.20)$	0.4794	[0.4163, 0.5409]	0.5147
λ_1	$\mathcal{N}(0.01, 0.20)$	0.0162	[0.0061, 0.0232]	0.0095
λ_2	$\mathcal{N}(0.001, 0.20)$	0.0011	[0.0004, 0.0019]	0.0006
ρ_y	$\mathcal{B}(0.80, 0.20)$	0.7219	[0.6414, 0.7993]	0.8611
δ_1	$\mathcal{N}(0.015, 0.01)$	0.0171	[0.0016, 0.0266]	0.0292
δ_2	$\mathcal{N}(0.002, 0.10)$	0.0027	[0.0012, 0.0034]	0.0058
δ_3	$\mathcal{N}(0.05, 0.10)$	0.0718	[0.0354, 0.1067]	0.0480
ρ_r	$\mathcal{B}(0.65, 0.20)$	0.6735	[0.6279, 0.7360]	0.6829
ψ_1	$\mathcal{N}(1.65, 0.20)$	1.6792	[1.5829, 1.7838]	1.2591
ψ_2	$\mathcal{N}(1.25, 0.20)$	1.2306	[1.0808, 1.3778]	1.8352
$\rho_{\epsilon_{IS}}$	$\mathcal{B}(0.60, 0.10)$	0.4522	[0.3114, 0.5793]	0.3980
$\rho_{\epsilon_{AS}}$	$\mathcal{B}(0.45, 0.20)$	0.7289	[0.7115, 0.7437]	0.9371
$\rho_{\epsilon_{RER}}$	$\mathcal{B}(0.70, 0.20)$	0.8818	[0.8365, 0.9121]	0.8806
σ_{IS}	$\mathcal{IG}(0.15, 0.20)$	0.1163	[0.0843, 0.1405]	0.0911
σ_{AS}	$\mathcal{IG}(0.10, 0.20)$	0.1848	[0.1602, 0.2085]	0.1738
σ_{MP}	$\mathcal{IG}(1.0, 0.20)$	0.8966	[0.7786, 1.0069]	0.8930
σ_{RER}	$\mathcal{IG}(4.0, 0.20)$	1.1623	[0.7577, 1.5973]	1.1683

Table 5: Comparison of MLE and Bayesian parameter estimates.

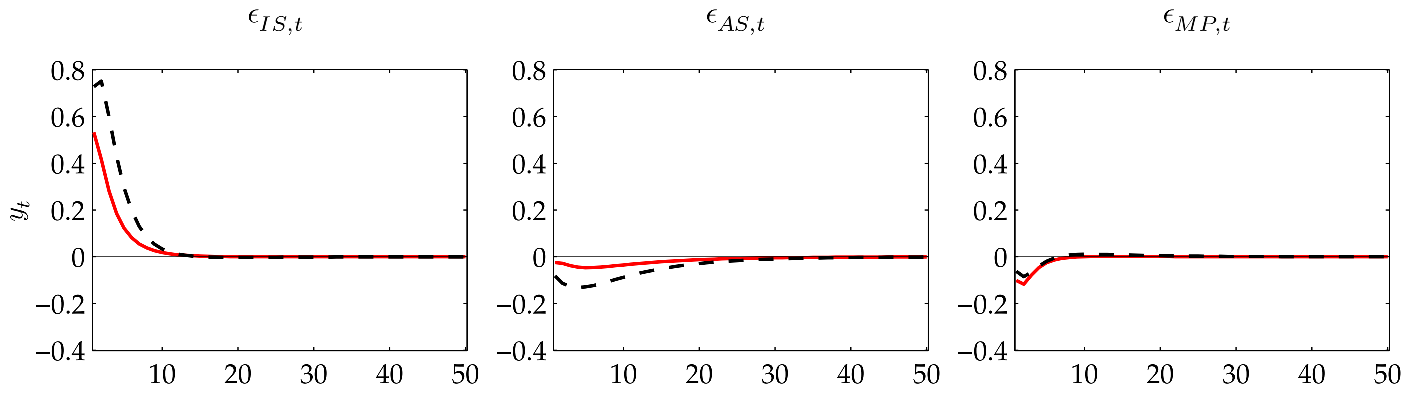


(a) Response of y_t to shocks in $\boldsymbol{\epsilon}^*_t$

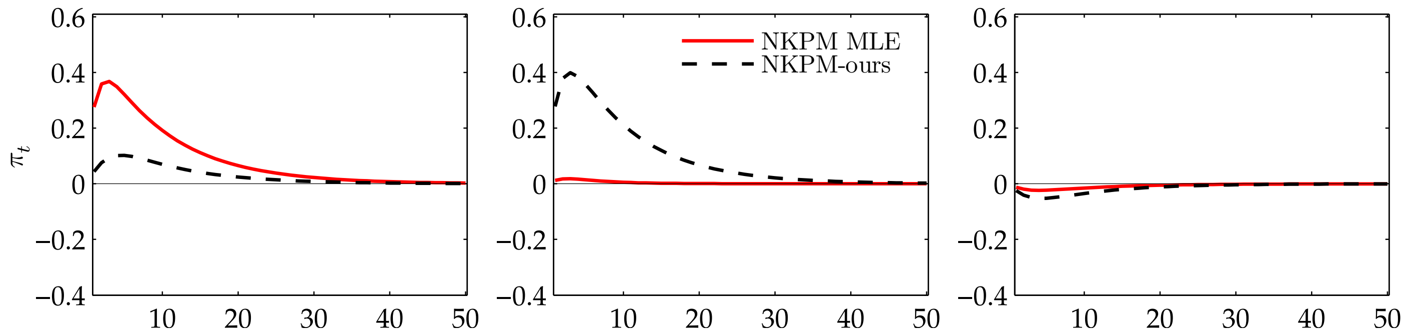
(b) Response of π_t to shocks in $\boldsymbol{\epsilon}^*_t$

(c) Response of r_t to shocks in $\boldsymbol{\epsilon}^*_t$

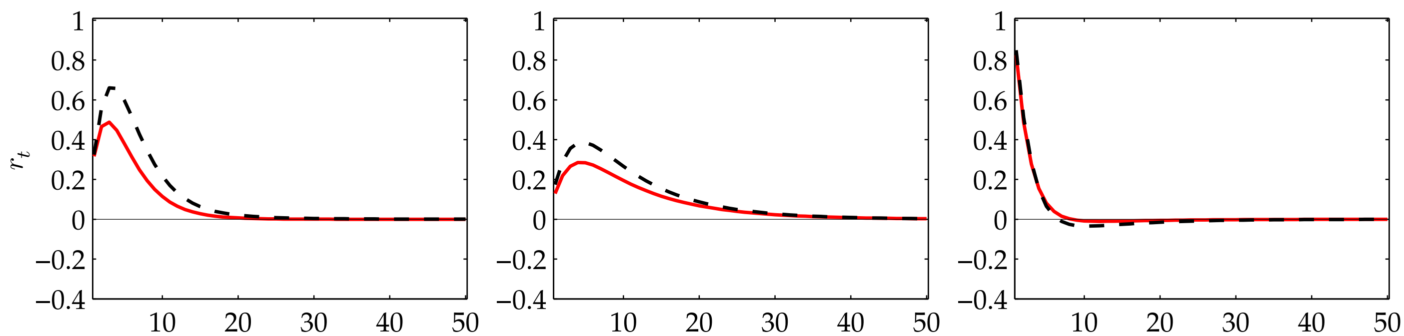
FIGURE 2: Comparison of IRFs under Cholesky and DP identification restrictions.



(a) Response of y_t to shocks in $\boldsymbol{\epsilon}_t$

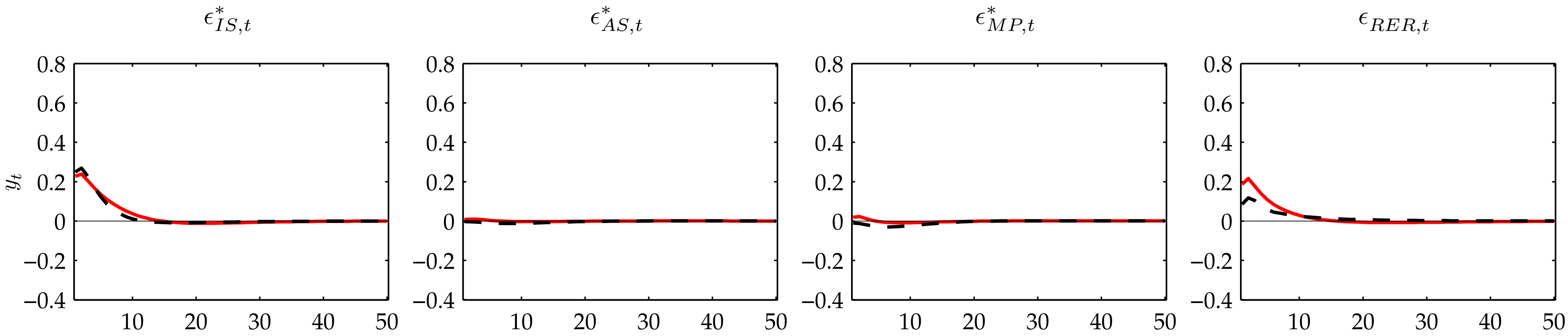


(b) Response of π_t to shocks in $\boldsymbol{\epsilon}_t$

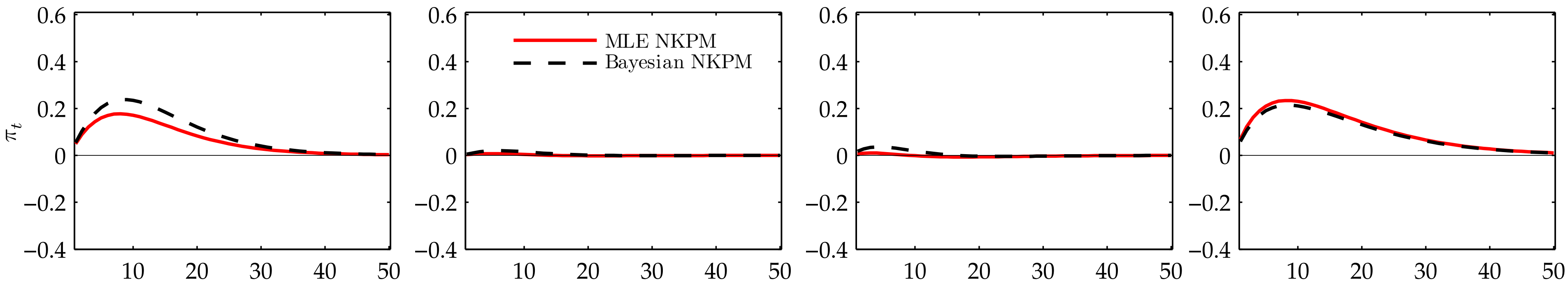


(c) Response of r_t to shocks in $\boldsymbol{\epsilon}_t$

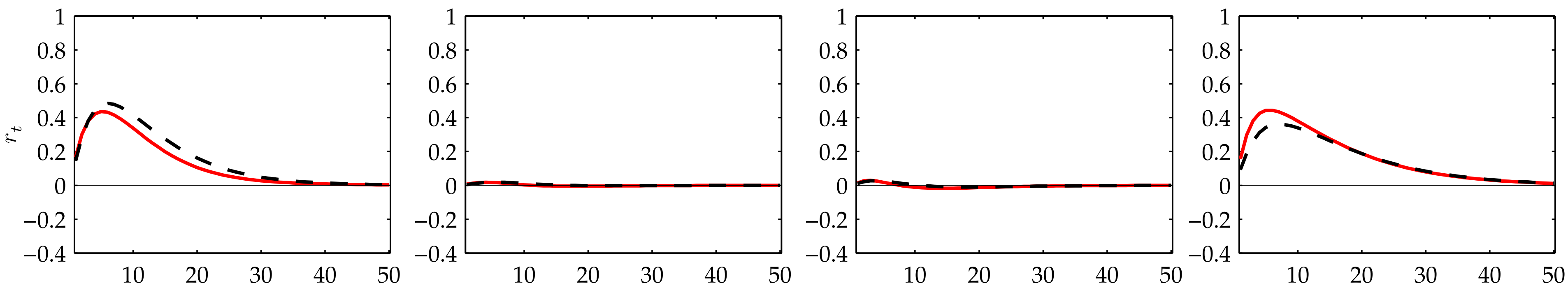
FIGURE 3: Comparison of MLE and Bayesian NKPM IRFs.



(a) Response of y_t to shocks in $\boldsymbol{\epsilon}_t^*$



(b) Response of π_t to shocks in $\boldsymbol{\epsilon}_t^*$



(c) Response of r_t to shocks in $\boldsymbol{\epsilon}_t^*$

FIGURE 4: Comparison of MLE and Bayesian NKPM IRFs.