LINEAR TIME SERIES ANALYSIS

UNIVERSITY OF ST. GALLEN

March 4, 2016

Homework: (20%)

Due Date: 17.04.2016

THE RULES: This is a **group project**. Form a group of $4-5$ people (**you must form a group**). The homework answers can be emailed to me. Put *"LTSA homework answers"* in the subject header of the email. **Due time is 00.00 (12am)** midnight to keep things fair for everyone.

NOTE: You can use any computer package that you like for this exercise, but I would recommend using R since we are using it in class. Also, you **must attach your clean and documented code** to the assignment when you hand it in.

It is your responsibility to contact me if something is not clear or you suspect an error.

A. Algebra

1) Let $\{X_t\}$ be the AR(1) process defined as:

$$X_t = \phi X_{t-1} + Z_t, \ \sim \mathsf{WN}(0, \sigma^2). \tag{1}$$

a) Compute the variance of the sample mean $\overline{X}_4$ when $\phi = 0.9$ and $\sigma^2 = 1$.

b) Repeat *a)* when $\phi = -0.9$ and compare your answer with the result in *a)*.

2) Let $\{Y_t\}$ be the **AR(1) plus noise** time series defined by

$$Y_t = X_t + W_t,$$

where $\{W_t\} \sim \mathsf{WN}(0, \sigma_W^2)$, $\{X_t\}$ is the AR(1) process

$$X_t - \phi X_{t-1} = Z_t, \ \{Z_t\} \sim \mathsf{WN}(0, \sigma_Z^2), \tag{2}$$

and $E[W_s Z_t] = 0$ for all s and t.

a) Show that $\{Y_t\}$ is stationary and find its autocovariance function.

b) Show that the time series $U_t = Y_t - \phi Y_{t-1}$ is $1-$correlated and hence an MA(1) process.

3) Show that the two MA(1) processes

$$Y_t = Z_t + \theta Z_{t-1}, \quad \{Z_t\} \sim \mathsf{WN}(0, \sigma^2)$$

and

$$Y_t = W_t + \frac{1}{\theta} W_{t-1}, \quad \{W_t\} \sim \mathsf{WN}(0, \sigma^2 \theta^2),$$

where $0 < |\theta| < 1$, have the same autocorrelation function (ACF).

4) Let X_t follow an ARMA process of the form

$$X_t + 0.6X_{t-1} - 0.3X_{t-2} = Z_t + 0.3Z_{t-1} - 0.5Z_{t-2} \tag{3}$$

where $\{Z_t\} \sim \mathsf{WN}(0, \sigma^2)$

a) Write the model in Lag operator form and determine if the process is stationary and/or invertible.

b) Compute the first 5 coefficients $\psi_0, \psi_1, \ldots, \psi_4$ of the MA(∞) representation of $\{X_t\}$.

B. Empirical Modelling

Use what you have learned in this course together with the quarterly US real GDP data available from: http://www.danielbuncic.com/data/usgdpc96.xls to do the following:

a) Compute annualised real GDP growth as $y_t = 400 \times \ln(GDP_t/GDP_{t-1})$.

b) Fit an ARMA model of your liking to y_t (annualised GDP growth). Discuss in detail why you selected this model and why you think it is appropriate. Describe the process that you used to select this model. Discuss any problems you encountered throughout the modelling of y_t.