Chapter 15
Panel Data Models

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- A panel of data consists of a group of cross-sectional units (people, households, firms, states, countries) who are observed over time
  - Denote the number of cross-sectional units (individuals) by N
  - Denote the number of time periods in which we observe them as T

- Different ways of describing panel data sets:
  - Long and narrow
    - "Long" describes the time dimension and "narrow" implies a relatively small number of cross sectional units
  - Short and wide
    - There are many individuals observed over a relatively short period of time
  - Long and wide
    - Both *N* and *T* are relatively large

- It is possible to have data that combines cross-sectional and time-series data which do not constitute a panel
  - We may collect a sample of data on individuals from a population at several points in time, but the individuals are not the same in each time period
    - Such data can be used to analyze a "natural experiment"

15.1
A Microeconomic Panel

- In microeconomic panels, the individuals are not always interviewed the same number of times, leading to an **unbalanced panel** in which the number of time series observations is different across individuals
  - In a **balanced panel**, each individual has the same number of observations

Table 15.1 Representative Observations from NLS Panel Data

ID	*YEAR*	*LWAGE*	*EDUC*	*SOUTH*	*BLACK*	*UNION*	*EXPER*	*TENURE*
1	82	1.8083	12	0	1	1	7.6667	7.6667
1	83	1.8634	12	0	1	1	8.5833	8.5833
1	85	1.7894	12	0	1	1	10.1795	1.8333
1	87	1.8465	12	0	1	1	12.1795	3.7500
1	88	1.8564	12	0	1	1	13.6218	5.2500
2	82	1.2809	17	0	0	0	7.5769	2.4167
2	83	1.5159	17	0	0	0	8.3846	3.4167
2	85	1.9302	17	0	0	0	10.3846	5.4167
2	87	1.9190	17	0	0	1	12.0385	0.3333
2	88	2.2010	17	0	0	1	13.2115	1.7500
3	82	1.8148	12	0	0	0	11.4167	11.4167
3	83	1.9199	12	0	0	1	12.4167	12.4167
3	85	1.9584	12	0	0	0	14.4167	14.4167
3	87	2.0071	12	0	0	0	16.4167	16.4167
3	88	2.0899	12	0	0	0	17.8205	17.7500

15.2
A Microeconomic Panel

- A **pooled model** is one where the data on different individuals are simply pooled together with no provision for individual differences that might lead to different coefficients

Eq. 15.1

$$y_{it} = \beta_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$$

– Notice that the coefficients (β_1, β_2, β_3) do not have *i* or *t* subscripts

- The least squares estimator, when applied to a pooled model, is referred to as **pooled least squares**
  - The data for different individuals are pooled together, and the equation is estimated using least squares

- It is useful to write explicitly the error assumptions required for pooled least squares to be consistent and for the *t* and *F* statistics to be valid when computed using the usual least squares variance estimates and standard errors

Eq. 15.2 $$E\left(e_{it}\right)=0$$

Eq. 15.3 $$\operatorname{var}\left(e_{it}\right)=E\left(e_{it}^2\right)=\sigma_e^2$$

Eq. 15.4 $$\operatorname{cov}\left(e_{it},e_{js}\right)=E\left(e_{it},e_{js}\right)=0 \text{ for } i\neq j \text{ or } t\neq s$$

Eq. 15.5 $$\operatorname{cov}\left(e_{it},x_{2it}\right)=0,\quad \operatorname{cov}\left(e_{it},x_{3it}\right)=0$$

- Applying pooled least squares in a way that ignores the panel nature of the data is restrictive in a number of ways
  - The first unrealistic assumption that we consider is the lack of correlation between errors corresponding to the same individual

- To relax the assumption of zero error correlation over time for the same individual, we write:

Eq. 15.6

$$\operatorname{cov}\left(e_{it}, e_{is}\right) = \psi_{ts}$$

  – This also relaxes the assumption of homoskedasticity:

$$\operatorname{cov}\left(e_{it}, e_{it}\right) = \operatorname{var}\left(e_{it}\right) = \psi_{tt}$$

- We continue to assume that the errors for different individuals are uncorrelated:

$$\operatorname{cov}\left(e_{it}, e_{js}\right) = 0 \text{ for } i \neq j$$

- What are the consequences of using pooled least squares in the presence of the heteroskedasticity and correlation?
  - The least squares estimator is still consistent
  - Its standard errors are incorrect
    - This implies that hypothesis tests and interval estimates based on these standard errors will be invalid
      - Typically, the standard errors will be too small, overstating the reliability of the least squares estimator

- Standard errors that are valid for the pooled least squares estimator under the assumption in Eq. 15.6 can be computed
  - Various names are:
    - Panel-robust standard errors
    - Cluster-robust standard errors
      - The time series observations on individuals are the clusters

Table 15.2 Pooled Least Squares Estimates of Wage Equation

		Least Squares Standard Errors			Cluster-Robust Standard Errors		
Variable	Coefficient	Std. Error	t-value	p-value	Std. Error	t-value	p-value
C	0.47660	0.05616	8.49	0.000	0.08456	5.64	0.000
$EDUC$	0.07145	0.00269	26.57	0.000	0.00550	12.99	0.000
$EXPER$	0.05569	0.00861	6.47	0.000	0.01130	4.92	0.000
$EXPER^2$	−0.00115	0.00036	−3.18	0.002	0.00049	−2.33	0.020
$TENURE$	0.01496	0.00441	3.39	0.001	0.00712	2.10	0.036
$TENURE^2$	−0.00049	0.00026	−1.89	0.059	0.00041	−1.18	0.236
$BLACK$	−0.11671	0.01572	−7.43	0.000	0.02813	−4.15	0.000
$SOUTH$	−0.10600	0.01420	−7.46	0.000	0.02706	−3.92	0.000
$UNION$	0.13224	0.01496	8.84	0.000	0.02707	4.88	0.000

15.3
The Fixed Effects Model

- We can extend the model in Eq. 15.1 to relax the assumption that all individuals have the same coefficients:

Eq. 15.7

$$y_{it} = \beta_{1i} + \beta_{2i} x_{2it} + \beta_{3i} x_{3it} + e_{it}$$

  – An i subscript has been added to each of the subscripts, implying that (β_1, β_2, β_3) can be different for each individual

- A popular simplification is one where the intercepts β_{1i} are different for different individuals but the slope coefficients β_2 and β_3 are assumed to be constant for all individuals:

Eq. 15.8

$$y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$$

- All behavioral differences between individuals, referred to as **individual heterogeneity**, are assumed to be captured by the intercept
  - Individual intercepts are included to “control” for individual-specific, time-invariant characteristics.
  - A model with these features is called a **fixed effects model**
    - The intercepts are called **fixed effects**

- We consider two methods for estimating Eq. 15.8
  1. The least squares dummy variable estimator
  2. The fixed effects estimator

- One way to estimate the model in Eq. 15.8 is to include an intercept dummy variable (indicator variable) for each individual

$$D_{1i} = \begin{cases} 1 & i=1 \\ 0 & \text{otherwise} \end{cases} \quad D_{2i} = \begin{cases} 1 & i=2 \\ 0 & \text{otherwise} \end{cases} \quad D_{3i} = \begin{cases} 1 & i=3 \\ 0 & \text{otherwise} \end{cases}$$

  – If we have 10 individuals, we define 10 such dummies

- Now we can write:

Eq. 15.9

$$y_{it} = \beta_{11} D_{1i} + \beta_{12} D_{2i} + \cdots + \beta_{1,10} D_{10i} + \beta_2 V_{2it} + \beta_3 K_{3it} + e_{it}$$

- If the error terms e_{it} are uncorrelated with mean zero and constant variance σ^2_e for all observations, then the best linear unbiased estimator of Eq. 15.9 is the least squares estimator
  - In a panel data context, it is called the **least squares dummy variable estimator**

Table 15.3 Dummy Variable Estimation of Wage Equation for $N = 10$

Variable	Coefficient	Std. Error	t-value	p-value
$D1$	0.1519	1.0967	0.139	0.891
$D2$	0.1869	1.0715	0.174	0.863
$D3$	−0.0630	1.3509	−0.047	0.963
$D4$	0.1856	1.3435	0.138	0.891
$D5$	0.9390	1.0978	0.855	0.398
$D6$	0.7945	1.1118	0.715	0.480
$D7$	0.5812	1.2359	0.470	0.641
$D8$	0.5379	1.0975	0.490	0.627
$D9$	0.4183	1.0840	0.386	0.702
$D10$	0.6146	1.0902	0.564	0.577
$EXPER$	0.2380	0.1878	1.268	0.213
$EXPER^2$	−0.0082	0.0079	−1.036	0.307
$TENURE$	−0.0124	0.0341	−0.362	0.720
$TENURE^2$	0.0023	0.0027	0.854	0.399
$UNION$	0.1135	0.1509	0.753	0.457

$SSE = 2.667190$

Table 15.4 Pooled Least Squares Estimates of Wage Equation for $N = 10$

Variable	Coefficient	Std. Error	t-value	p-value
C	0.6209	1.0172	0.610	0.545
$EXPER$	0.1947	0.1730	1.125	0.267
$EXPER^2$	−0.0049	0.0071	−0.688	0.495
$TENURE$	0.0014	0.0375	0.036	0.971
$TENURE^2$	−0.0009	0.0023	−0.371	0.712
$UNION$	−0.0175	0.1024	−0.171	0.865

$SSE = 5.502466.$

- We can test the estimates of the intercepts:

Eq. 15.10

$$H_0 : \beta_{11} = \beta_{12} = \cdots = \beta_{1,10}$$

$$H_1 : \text{the } \beta_{1i} \text{ are not all equal}$$

- These *N*-1 = 9 joint null hypotheses are tested using the usual *F*-test statistic
  - In the restricted model all the intercept parameters are equal
  - If we call their common value β_1, then the restricted model is the pooled model:

$$\ln(WAGE) = \beta_1 + \beta_2 EXPER + \beta_3 EXPER^2 + \beta_4 TENURE + \beta_5 TENURE^2 + \beta_6 UNION + e$$

- The *F*-statistic is:

$$F = \frac{\left(SSE_R - SSE_U\right)/J}{SSE_U/\left(NT - K\right)} = \frac{\left(5.502466 - 2.667190\right)/9}{2.667190/\left(50 - 15\right)} = 4.134$$

- The value of the test statistic $F = 4.134$ yields a *p*-value of 0.0011
  - We reject the null hypothesis that the intercept parameters for all individuals are equal.
  - We conclude that there are differences in individual intercepts, and that the data should not be pooled into a single model with a common intercept parameter

- Using the dummy variable approach is not feasible when N is large
  - Another approach is necessary

- Take the data on individual *i*:

Eq. 15.11

$$y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it} \qquad t = 1, \ldots, T$$

– Average the data across time:

$$\frac{1}{T}\sum_{t=1}^{T}\left(y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}\right)$$

- Using the fact that the parameters do not change over time, we can simplify this as:

Eq. 15.12

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it} = \beta_{1i} + \beta_2 \frac{1}{T}\sum_{t=1}^{T} x_{2it} + \beta_3 \frac{1}{T}\sum_{t=1}^{T} x_{3it} + \frac{1}{T}\sum_{t=1}^{T} e_{it}$$

$$= \beta_{1i} + \beta_2 \bar{x}_{2i} + \beta_3 \bar{x}_{3i} + \bar{e}_i$$

15.3.2
The Fixed Effects Estimator

- Now subtract Eq. 15.12 from Eq. 15.11, term by term, to obtain:

$$y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$$

$$- \quad (\bar{y}_i = \beta_{1i} + \beta_2 \bar{x}_{2i} + \beta_3 \bar{x}_{3i} + \bar{e}_i)$$

Eq. 15.13

$$y_{it} - \bar{y}_i = \beta_2 (x_{2it} - \bar{x}_{2i}) + \beta_3 (x_{3it} - \bar{x}_{3i}) + (e_{it} - \bar{e}_i)$$

or

Eq. 15.14

$$\tilde{y}_{it} = \beta_2 \tilde{x}_{it} + \beta_3 \tilde{x}_{it} + \tilde{e}_{it}$$

Table 15.5 Data in Deviation from Individual Mean Form

i	t	y_{it}	$\bar{y}_i$	$\tilde{y}_{it} = y_{it} - \bar{y}_i$	x_{2it}	$\bar{x}_{2i}$	$\tilde{x}_{2it} = x_{2it} - \bar{x}_{2i}$
1	1	1.8083	1.8328	−0.0245	7.667	10.446	−2.779
1	2	1.8634	1.8328	0.0306	8.583	10.446	−1.863
1	3	1.7894	1.8328	−0.0434	10.179	10.446	−0.267
1	4	1.8465	1.8328	0.0137	12.179	10.446	1.733
1	5	1.8564	1.8328	0.0236	13.622	10.446	3.176
2	1	1.2809	1.7694	−0.4885	7.577	10.319	−2.742
2	2	1.5159	1.7694	−0.2535	8.385	10.319	−1.935
2	3	1.9302	1.7694	0.1608	10.385	10.319	0.065
2	4	1.9190	1.7694	0.1496	12.038	10.319	1.719
2	5	2.2010	1.7694	0.4316	13.212	10.319	2.892
3	1	1.8148	1.9580	−0.1432	11.417	14.497	−3.081
3	2	1.9199	1.9580	−0.0381	12.417	14.497	−2.081
3	3	1.9584	1.9580	0.0004	14.417	14.497	−0.081
3	4	2.0071	1.9580	0.0491	16.417	14.497	1.919
3	5	2.0899	1.9580	0.1318	17.821	14.497	3.323

$y_{it} = LWAGE_{it}, \quad x_{2it} = EXPER_{it}.$

Table 15.6 Fixed Effects Estimation of Wage Equation for $N = 10$

	Using Least Squares Deviation Form		Using Fixed Effects Software Command	
Variable	Coefficient	Std. Error	Coefficient	Std. Error
C			0.4347	1.1452
$EXPER$	0.2380	0.1656	0.2380	0.1878
$EXPER^2$	–0.0082	0.0070	–0.0082	0.0079
$TENURE$	–0.0124	0.0301	–0.0124	0.0341
$TENURE^2$	0.0023	0.0024	0.0023	0.0027
$UNION$	0.1135	0.1330	0.1135	0.1509
	$SSE = 2.66719$		$SSE = 2.66719$	

- If we multiply the standard errors from estimating Eq. 15.4 by the correction factor

$$\sqrt{(NT-5)/(NT-N-5)} = \sqrt{45/35} = 1.133893$$

the resulting standard errors are identical to those in Table 15.3

- Usually we are most interested in the coefficients of the explanatory variables and not the individual intercept parameters
  - These coefficients can be "recovered" by using the fact that the least squares fitted regression passes through the point of the means
  - That is:

$$\bar{y}_i = b_{1i} + b_2 \bar{x}_{2i} + b_3 \bar{x}_{3i}$$

  - So that the fixed effects are:

Eq. 15.15

$$b_{1i} = \bar{y}_i - b_2 \bar{x}_{2i} - b_3 \bar{x}_{3i} \quad i = 1, \ldots, N$$

Table 15.7 Fixed Effects Estimates of Wage Equation for $N = 716$

		Least Squares Standard Errors			Cluster-Robust Standard Errors		
Variable	Coefficient	Std. Error	t-value	p-value	Std. Error	t-value	p-value
C	1.45003	0.04014	36.12	0.000	0.06153	23.57	0.000
$EXPER$	0.04108	0.00662	6.21	0.000	0.00921	4.46	0.000
$EXPER^2$	–0.00041	0.00027	–1.50	0.135	0.00037	–1.11	0.268
$TENURE$	0.01391	0.00328	4.24	0.000	0.00471	2.95	0.003
$TENURE^2$	–0.00090	0.00021	–4.35	0.000	0.00028	–3.21	0.001
$SOUTH$	–0.01632	0.03615	–0.45	0.652	0.06539	–0.25	0.803
$UNION$	0.06370	0.01425	4.47	0.000	0.01885	3.38	0.001

Table 15.8 Percentage Marginal Effects on Wages

Variable	Pooled Least Squares	Fixed Effects Estimator
EXPER	2.81	3.13
TENURE	0.82	0.14
SOUTH	–10.60	–1.63
UNION	13.22	6.37

15.4
The Random Effects Model

- In the random effects model we assume that all individual differences are captured by the intercept parameters
  - But we also recognize that the individuals in our sample were randomly selected, and thus we treat the individual differences as random rather than fixed, as we did in the fixed-effects dummy variable model

- Random individual differences can be included in our model by specifying the intercept parameters to consist of a fixed part that represents the population average and random individual differences from the population average:

Eq. 15.16

$$\beta_{1i} = \overline{\beta}_1 + u_i$$

  – The random individual differences u_i are called **random effects** and have:

Eq. 15.17

$$E(u_i) = 0, \quad \text{cov}(u_i, u_j) = 0, \quad \text{var}(u_i) = \sigma_u^2$$

- Substituting, we get:

Eq. 15.18

$$y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$$

$$= \left(\overline{\beta}_1 + u_i\right) + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$$

- Rearranging:

Eq. 15.19

$$y_{it} = \overline{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + \left(e_{it} + u_i\right)$$

$$= \overline{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + v_{it}$$

- The combined error term is:

Eq. 15.20

$$v_{it} = u_i + e_{it}$$

  - The random effects error has two components:
    - One for the individual
    - One for the regression
  - The random effects model is often called an **error components model**

- The combined error term has zero mean:

$$E(v_{it}) = E(u_i + e_{it}) = E(u_i) + E(e_{it}) = 0 + 0 = 0$$

  – And a constant, homoskedastic, variance:

$$\sigma_v^2 = \operatorname{var}(v_{it}) = \operatorname{var}(u_i + e_{it})$$

Eq. 15.21

$$= \operatorname{var}(u_i) + \operatorname{var}(e_{it}) + 2\operatorname{cov}(u_i, e_{it})$$

$$= \sigma_u^2 + \sigma_e^2$$

- There are several correlations that can be considered:

  1. The correlation between two individuals, *i* and *j*, at the same point in time, *t*.

$$\operatorname{cov}\left(v_{it}, v_{jt}\right) = E(v_{it} v_{jt}) = E\left[\left(u_i + e_{it}\right)\left(u_j + e_{jt}\right)\right]$$

$$= E\left(u_i u_j\right) + E\left(u_i e_{jt}\right) + E\left(e_{it} u_j\right) + E\left(e_{it} e_{jt}\right)$$

$$= 0 + 0 + 0 + 0 = 0$$

- There are several correlations (Continued):

    2. The correlation between errors on the same individual (i) at different points in time, t and s

Eq. 15.22

$$\text{cov}(v_{it}, v_{is}) = E(v_{it}v_{is}) = E\left[(u_i + e_{it})(u_i + e_{is})\right]$$

$$= E(u_i^2) + E(u_i e_{is}) + E(e_{it}u_i) + E(e_{it}e_{is})$$

$$= \sigma_u^2 + 0 + 0 + 0$$

$$= \sigma_u^2$$

- There are several correlations (Continued):

  3. The correlation between errors for different individuals in different time periods

$$\operatorname{cov}\left(v_{it}, v_{js}\right) = E(v_{it} v_{js}) = E\left[\left(u_i + e_{it}\right)\left(u_j + e_{js}\right)\right]$$

$$= E\left(u_i u_j\right) + E\left(u_i e_{js}\right) + E\left(e_{it} u_j\right) + E\left(e_{it} e_{js}\right)$$

$$= 0 + 0 + 0 + 0 = 0$$

15.4.1
Error Term Assumptions

- The errors $v_{it} = u_i + e_{it}$ are correlated over time for a given individual, but are otherwise uncorrelated
  - The correlation is caused by the component u_i that is common to all time periods
  - It is constant over time and, in contrast to the AR(1) error model, it does not decline as the observations get further apart in time:

Eq. 15.23

$$\rho = \text{corr}(v_{it}, v_{is}) = \frac{\text{cov}(v_{it}, v_{is})}{\sqrt{\text{var}(v_{it})\,\text{var}(v_{is})}} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} \qquad t \neq s$$

- In terms of the notation introduced to explain the assumptions that motivate the use of cluster-robust standard errors:

$$\text{var}(v_{it}) = \psi_{it} = \sigma_u^2 + \sigma_e^2 \quad \text{and} \quad \text{cov}(v_{it}, v_{is}) = \psi_{is} = \sigma_u^2 \quad t \neq s$$

- Summary of the error term assumptions of the random effects model:

Eq. 15.24 $$E\left(v_{it}\right) = 0$$

Eq. 15.25 $$\text{var}(v_{it}) = \sigma_u^2 + \sigma_e^2$$

Eq. 15.26 $$\text{cov}(v_{it}, v_{is}) = \sigma_u^2 \quad t \neq s$$

Eq. 15.27 $$\text{cov}(v_{it}, v_{js}) = 0 \quad i \neq j$$

Eq. 15.28 $$\text{cov}(e_{it}, x_{2it}) = 0, \quad \text{cov}(e_{it}, x_{3it}) = 0$$

Eq. 15.29 $$\text{cov}(u_i, x_{2it}) = 0, \quad \text{cov}(u_i, x_{3it}) = 0$$

- We can test for the presence of heterogeneity by testing the null hypothesis H_0: $\sigma^2_u = 0$ against the alternative hypothesis H_1: $\sigma^2_u > 0$
  - If the null hypothesis is rejected, then we conclude that there are random individual differences among sample members, and that the random effects model is appropriate
  - If we fail to reject the null hypothesis, then we have no evidence to conclude that random effects are present

- The Lagrange multiplier (LM) principle for test construction is very convenient in this case
  - If the null hypothesis is true, then $u_\text{i} = 0$ and the random effects model in Eq. 15.19 reduces to:

$$y_{it} = \overline{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$$

- The test statistic is based on the least squares residuals:

$$\hat{e}_{it} = y_{it} - \bar{b}_1 - b_2 x_{2it} - b_3 x_{3it}$$

  – The test statistic for balanced panels is:

Eq. 15.30

$$LM = \sqrt{\frac{NT}{2(T-1)}} \left\{ \frac{\sum_{i=1}^{N} \left(\sum_{t=1}^{T} \hat{e}_{it} \right)^2}{\sum_{i=1}^{N} \sum_{t=1}^{T} \hat{e}_{it}^2} - 1 \right\}$$

- If the null hypothesis H_0: $\sigma^2_u = 0$ is true, then LM ~ $N(0, 1)$ in large samples
  - Thus, we reject H_0 at significance level α and accept the alternative H_1: $\sigma^2_u > 0$ if LM > $z_{(1-\alpha)}$, where $z_{(1-\alpha)}$ is the 100(1–α) percentile of the standard normal distribution
  - This critical value is 1.645 if α = 0.05 and 2.326 if α = 0.01
  - Rejecting the null hypothesis leads us to conclude that random effects are present

- We can obtain the generalized least squares estimator in the random effects model by applying least squares to a transformed model:

Eq. 15.31

$$y_{it}^{*} = \overline{\beta}_1 x_{1it}^{*} + \beta_2 x_{2it}^{*} + \beta_3 x_{3it}^{*} + v_{it}^{*}$$

where the transformed variables are:

Eq. 15.32

$$y_{it}^{*} = y_{it} - \alpha \overline{y}_i, \quad x_{1it}^{*} = 1 - \alpha, \quad x_{2it}^{*} = x_{2it} - \alpha \overline{x}_{2i}, \quad x_{3it}^{*} = x_{3it} - \alpha \overline{x}_{3i}$$

and α is defined as

Eq. 15.33

$$\alpha = 1 - \frac{\sigma_e}{\sqrt{T\sigma_u^2 + \sigma_e^2}}$$

- For $\alpha = 1$, the random effects estimator is identical to the fixed effects estimator
- For $\alpha < 1$, it can be shown that the random effects estimator is a "matrix-weighted average" of the fixed effects estimator that utilizes only within individual variation and a "between estimator" which utilizes variation between individuals

Table 15.9 Random Effects Estimates of Wage Equation

		GLS Standard Errors[a]			Cluster-Robust Standard Errors[a]		
Variable	Coefficient	Std. Error	*t*-value	*p*-value	Std. Error	*t*-value	*p*-value
C	0.53393	0.07988	6.68	0.000	0.08209	6.50	0.000
EDUC	0.07325	0.00533	13.74	0.000	0.00540	13.57	0.000
EXPER	0.04362	0.00636	6.86	0.000	0.00755	5.78	0.000
$EXPER^2$	–0.00056	0.00026	–2.14	0.033	0.00031	–1.83	0.068
TENURE	0.01415	0.00317	4.47	0.000	0.00400	3.54	0.000
$TENURE^2$	–0.00076	0.00019	–3.88	0.000	0.00024	–3.21	0.001
BLACK	–0.11674	0.03021	–3.86	0.000	0.02928	–3.99	0.000
SOUTH	–0.08181	0.02241	–3.65	0.000	0.02833	–2.89	0.004
UNION	0.08024	0.01321	6.07	0.000	0.01547	5.19	0.000

[a]Different software can give standard errors with very slight differences. Those reported are from Stata Version 11.0.

The estimate of the transformation parameter α is:

$$\hat{\alpha} = 1 - \frac{\hat{\sigma}_e}{\sqrt{T\hat{\sigma}_u^2 + \hat{\sigma}_e^2}} = 1 - \frac{0.1951}{\sqrt{5 \times 0.1083 + 0.0381}} = 0.7437$$

15.5
Comparing Fixed and Random Effects Estimators

- If random effects are present, then the random effects estimator is preferred for several reasons:
  1. The random effects estimator takes into account the random sampling process by which the data were obtained
  2. The random effects estimator permits us to estimate the effects of variables that are individually time-invariant
  3. The random effects estimator is a generalized least squares estimation procedure, and the fixed effects estimator is a least squares estimator

- If the random error $v_{it} = u_i + e_{it}$ is correlated with any of the right-hand-side explanatory variables in a random effects model, then the least squares and GLS estimators of the parameters are biased and inconsistent
  - The problem of **endogenous regressors** was considered before
  - The problem is common in random effects models, because the individual specific error component u_i may well be correlated with some of the explanatory variables

- The panel data regression Eq. 15.19, including the error component u_i, is:

Eq. 15.34

$$y_{it} = \overline{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + (u_i + e_{it})$$

- Average the observations for each individual over time:

Eq. 15.35

$$\overline{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it} = \overline{\beta}_1 + \beta_2 \frac{1}{T}\sum_{t=1}^{T} x_{2it} + \beta_3 \frac{1}{T}\sum_{t=1}^{T} x_{3it} + \frac{1}{T}\sum_{t=1}^{T} u_i + \frac{1}{T}\sum_{t=1}^{T} e_{it}$$

$$= \overline{\beta}_1 + \beta_2 \overline{x}_{2i} + \beta_3 \overline{x}_{3i} + u_i + \overline{e}_i$$

- Subtract:

Eq. 15.36

$$y_{it} = \bar{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + u_i + e_{it}$$

$$- \quad (\bar{y}_i = \bar{\beta}_1 + \beta_2 \bar{x}_{2i} + \beta_3 \bar{x}_{3i} + u_i + \bar{e}_i)$$

$$y_{it} - \bar{y}_i = \beta_2 (x_{2it} - \bar{x}_{2i}) + \beta_3 (x_{3it} - \bar{x}_{3i}) + (e_{it} - \bar{e}_i)$$

- To check for any correlation between the error component u_i and the regressors in a random effects model, we can use a Hausman test
  - The Hausman test can be carried out for specific coefficients, using a t-test, or jointly, using an F-test or a chi-square test

- Let the parameter of interest be β_k
  - Denote the fixed effects estimate as $b_{FE,k}$ and the random effects estimate as $b_{RE,k}$
  - The *t*-statistic for testing that there is no difference between the estimators is:

Eq. 15.37

$$t = \frac{b_{FE,k} - b_{RE,k}}{\left[\widehat{\text{var}}\left(b_{FE,k}\right) - \widehat{\text{var}}\left(b_{RE,k}\right)\right]^{1/2}} = \frac{b_{FE,k} - b_{RE,k}}{\left[\text{se}\left(b_{FE,k}\right)^2 - \text{se}\left(b_{RE,k}\right)^2\right]^{1/2}}$$

- We expect to find:

$$\widehat{\mathrm{var}}\left(b_{FE,k}\right)-\widehat{\mathrm{var}}\left(b_{RE,k}\right)>0$$

  - Also:

$$\mathrm{var}\left(b_{FE,k}-b_{RE,k}\right)=\mathrm{var}\left(b_{FE,k}\right)+\mathrm{var}\left(b_{RE,k}\right)-2\,\mathrm{cov}\left(b_{FE,k},b_{RE,k}\right)$$

$$=\mathrm{var}\left(b_{FE,k}\right)-\mathrm{var}\left(b_{RE,k}\right)$$

because Hausman proved that:

$$\mathrm{cov}\left(b_{FE,k},b_{RE,k}\right)=\mathrm{var}\left(b_{RE,k}\right)$$

- Applying the *t*-test to the *SOUTH* we get:

$$t = \frac{b_{FE,k} - b_{RE,k}}{\left[\text{se}\left(b_{FE,k}\right)^2 - \text{se}\left(b_{RE,k}\right)^2\right]^{1/2}} = \frac{-0.01632 - (-0.08181)}{\left[(0.03615)^2 - (0.02241)^2\right]^{1/2}} = 2.31$$

  – Using the standard 5% large sample critical value of 1.96, we reject the hypothesis that the estimators yield identical results

    - Our conclusion is that the random effects estimator is inconsistent, and that we should use the fixed effects estimator, or should attempt to improve the model specification

- The form of the Hausman test in Eq. 15.37 and its χ^2 equivalent are not valid for cluster robust standard errors, because under these more general assumptions, it is no longer true that:

$$\operatorname{var}\left(b_{FE,k} - b_{RE,k}\right) = \operatorname{var}\left(b_{FE,k}\right) - \operatorname{var}\left(b_{RE,k}\right)$$

15.6
The Hausman-Taylor Estimator

- The Hausman-Taylor estimator is an instrumental variables estimator applied to the random effects model to overcome the problem of inconsistency caused by correlation between the random effects and some of the explanatory variables

- Consider the regression model:

Eq. 15.38

$$y_{it} = \beta_1 + \beta_2 x_{it,exog} + \beta_3 x_{it,endog} + \beta_4 w_{i,exog} + \beta_5 w_{i,endog} + u_i + e_{it}$$

with:

$x_{\text{it,exog}}$:exogenous variables that vary over time and individuals

$x_{\text{it,endog}}$:endogenous variables that vary over time and individuals

$w_{\text{i,exog}}$:time-invariant exogenous variables

$w_{\text{i,endog}}$:time-invariant endogenous variables

- A slightly modify set is applied to the transformed generalized least squares model from Eq. 15.31:

Eq. 15.39

$$y_{it}^{*} = \beta_1 + \beta_2 x_{it,exog}^{*} + \beta_3 x_{it,endog}^{*} + \beta_4 w_{i,exog}^{*} + \beta_5 w_{i,endog}^{*} + v_{it}^{*}$$

Table 15.10 Hausman-Taylor Estimates of Wage Equation

Variable	Coefficient	Std. Error	t-value	p-value
C	−0.75077	0.58624	−1.28	0.200
$EDUC$	0.17051	0.04446	3.83	0.000
$EXPER$	0.03991	0.00647	6.16	0.000
$EXPER^2$	−0.00039	0.00027	−1.46	0.144
$TENURE$	0.01433	0.00316	4.53	0.000
$TENURE^2$	−0.00085	0.00020	−4.32	0.000
$BLACK$	−0.03591	0.06007	−0.60	0.550
$SOUTH$	−0.03171	0.03485	−0.91	0.363
$UNION$	0.07197	0.01345	5.35	0.000

15.7 Sets of Regression Equations

- Consider procedures for a panel that is long and narrow: T is large relative to N
  - If the number of time series observations is sufficiently large, and N is small, we can estimate separate equations for each individual
  - These separate equations can be specified as

Eq. 15.40

$$y_{it} = \beta_{1i} + \beta_{2i} x_{2it} + \beta_{3i} x_{3it} + e_{it}$$

- An economic model for describing gross firm investment for the *i*th firm in the *t*th time period, denoted INV_{it}, may be expressed as:

Eq. 15.41

$$INV_{it} = f\left(V_{it}, K_{it}\right)$$

- We specify the following two equations for General Electric and Westinghouse:

Eq. 15.42

$$INV_{GE,t} = \beta_1 + \beta_2 V_{GE,t} + \beta_3 K_{GE,t} + e_{GE,t} \qquad t = 1935,\ldots,1954$$

$$INV_{WE,t} = \beta_1 + \beta_2 V_{WE,t} + \beta_3 K_{WE,t} + e_{WE,t} \qquad t = 1935,\ldots,1954$$

- The choice of estimator depends on what assumptions we make about the coefficients and the error terms:
  1. Are the *GE* coefficients equal to the *WE* coefficients?
  2. Do the equation errors $e_{GE,t}$ and $e_{WE,t}$ have the same variance?
  3. Are the equation errors $e_{GE,t}$ and $e_{WE,t}$ correlated?

- The assumption that both firms have the same coefficients and the same error variances can be written as:

Eq. 15.43

$$\beta_{1,GE} = \beta_{1,WE} \quad \beta_{2,GE} = \beta_{2,WE} \quad \beta_{3,GE} = \beta_{3,WE} \quad \sigma^2_{GE} = \sigma^2_{WE}$$

- Let D_i be an indicator variable equal to one for the Westinghouse observations and zero for the General Electric observations
  - Specify a model with slope and intercept indicator variables:

Eq. 15.44

$$INV_{it} = \beta_{1,GE} + \delta_1 D_i + \beta_{2,GE} V_{it} + \delta_2 \left(D_i \times V_{it} \right) + \beta_{3,GE} K_{it} + \delta_3 \left(D_i \times K_{it} \right) + e_{it}$$

Table 15.12 Least Squares Estimates from the Dummy Variable Model

Variable	Coefficient	Std. Error	t-value	p-value
C	−9.9563	23.6264	−0.42	0.676
D	9.4469	28.8054	0.33	0.745
V	0.0266	0.0117	2.27	0.030
$D \times V$	0.0263	0.0344	0.77	0.448
K	0.1517	0.0194	7.84	0.000
$D \times K$	−0.0593	0.1169	−0.51	0.615

$SSE = 14989.82$ $\hat{\sigma}^2 = 440.877$

- Using the Chow test, we get:

Eq. 15.45

$$F = \frac{\left(SSE_R - SSE_U\right)/J}{SSE_U/\left(NT - NK\right)} = \frac{\left(16563.00 - 14989.82\right)/3}{14989.82/\left(40-6\right)} = 1.189$$

where *NT* - *NK* is the total number of degrees of freedom in the unrestricted model

– The *p*-value for an $F_{(3,34)}$-distribution is 0.328, implying that the null hypothesis of equal coefficients cannot be rejected

- When both the coefficients and the error variances of the two equations differ, and in the absence of contemporaneous correlation that we introduce in the next section, there is no connection between the two equations, and the best we can do is apply least squares to each equation separately

Table 15.13 Least Squares Estimates of Separate Investment Equations

Equation	Variable	Coefficient	Std. Error	t-value	p-value
	C	−9.9563	31.3743	−0.32	0.755
GE	V	0.0266	0.0156	1.71	0.106
	K	0.1517	0.0257	5.90	0.000
	$SSE = 13216.59$		$\hat{\sigma}^2_{GE} = 777.446$		
	C	−0.5094	8.0153	−0.06	0.950
WE	V	0.0529	0.0157	3.37	0.004
	K	0.0924	0.0561	1.65	0.118
	$SSE = 1773.23$		$\hat{\sigma}^2_{WE} = 104.308$		

- Consider the following assumption:

Eq. 15.46

$$\operatorname{cov}\left(e_{GE,t}, e_{WE,t}\right) = \sigma_{GE,WE} \quad \sigma_{GE,WE} \neq 0$$

  – The error terms in the two equations, at the same point in time, are correlated
    - This kind of correlation is called **contemporaneous correlation**

- The dummy-variable model Eq. 15.44 represents a way to "stack" the 40 observations for the GE and WE equations into one regression
  - To improve the precision of the dummy variable model estimates, we use **seemingly unrelated regressions** (SUR) estimation, which is a generalized least squares estimation procedure
    - It estimates the two investment equations jointly, accounting for the fact that the variances of the error terms are different for the two equations and accounting for the contemporaneous correlation between the errors of the GE and WE equations

- Three stages in the SUR estimation procedure:
  1. Estimate the equations separately using OLS
  2. Use the OLS residuals from (1) to estimate σ^2_{GE}, σ^2_{WE} and $\sigma_{GE,WE}$
     - The estimated covariance is given by:

$$\hat{\sigma}_{GE,WE} = \frac{1}{\sqrt{T-K_{GE}}\sqrt{T-K_{WE}}}\sum_{t=1}^{20}\hat{e}_{GE,t}\hat{e}_{WE,t} = \frac{1}{T-3}\sum_{t=1}^{20}\hat{e}_{GE,t}\hat{e}_{WE,t}$$

$$= 207.587$$

  3. Use the estimates from (2) to estimate the two equations jointly within a generalized least squares framework

- The SUR estimation procedure is optimal under the contemporaneous correlation assumption, so no standard error adjustment is necessary

Table 15.14 SUR Estimates of Investment Equations

Equation	Variable	Coefficient	Std. Error	t-values	p-values
	C	–27.7193	29.3212	–0.95	0.351
GE	V	0.0383	0.0144	2.66	0.012
	K	0.1390	0.0250	5.56	0.000
	C	–1.2520	7.5452	–0.17	0.869
WE	V	0.0576	0.0145	3.96	0.000
	K	0.0640	0.0530	1.21	0.236

Note: p-values computed from $t_{(34)}$ distribution.

- Two situations in which separate least squares estimation is just as good as the SUR technique
  1. The equation errors are not contemporaneously correlated
     - If the errors are not contemporaneously correlated, there is nothing linking the two equations, and separate estimation cannot be improved upon
  2. Least squares and SUR give identical estimates when the same explanatory variables appear in each equation

- If the explanatory variables in each equation are different, then a test to see if the correlation between the errors is significantly different from zero is of interest
  - Compute the squared correlation:

$$r_{GE,WE}^2 = \frac{\hat{\sigma}_{GE,WE}^2}{\hat{\sigma}_{GE}^2 \hat{\sigma}_{WE}^2} = \frac{(207.5871)^2}{(777.4463)(104.3079)} = 0.5314$$

- To check the statistical significance of $r^2_{GE,WE}$, test the null hypothesis H_0: $\sigma_{GE,WE} = 0$
  - If $\sigma_{GE,WE} = 0$, then $LM = T \times r^2_{GE,WE}$ is a Lagrange Multiplier test statistic that is distributed as a $\chi^2_{(1)}$ random variable in large samples
  - The 5% critical value of a χ^2-distribution with one degree of freedom is 3.841
  - The value of the test statistic is $LM = 10.628$
  - We reject the null hypothesis of no correlation

- If we are testing for the existence of correlated errors for more than two equations, the relevant test statistic is equal to T times the sum of squares of all the correlations
  - The probability distribution under H_0 is a χ^2-distribution with degrees of freedom equal to the number of correlations

- With three equations, denoted by subscripts 1, 2 and 3, the null hypothesis is:

$$H_0: \ \sigma_{12} = \sigma_{13} = \sigma_{23} = 0$$

  - The $\chi^2_{(3)}$ test statistic is:

$$LM = T\left(r_{12}^2 + r_{13}^2 + r_{23}^2\right)$$

  - With M equations:

$$LM = T\sum_{i=2}^{M}\sum_{j=1}^{i-1} r_{ij}^2$$

with $M(M - 1)/2$ degrees of freedom

- We previously used the dummy variable model and the Chow test to test whether the two equations had identical coefficients:

Eq. 15.47

$$H_0 : \beta_{1,GE} = \beta_{1,WE} \quad \beta_{2,GE} = \beta_{2,WE} \quad \beta_{3,GE} = \beta_{3,WE}$$

  – It is also possible to test hypotheses such as Eq 15.47 when the more general error assumptions of the SUR model are relevant
    - Because of the complicated nature of the model, the test statistic can no longer be calculated simply as an F-test statistic based on residuals from restricted and unrestricted models

- Most econometric software will perform an *F*-test and/or a Wald χ^2-test in a multi-equation framework such as we have here
  - In the context of SUR equations both tests are large sample approximate tests

- The equality of coefficients is not the only cross-equation hypothesis that can be tested
  - Any restrictions on parameters in different equations can be tested
  - Tests for hypotheses involving coefficients within each equation are valid whether done on each equation separately or using the SUR framework
  - However, tests involving cross-equation hypotheses need to be carried out within an SUR framework if contemporaneous correlation exists

Key Words

Keywords

- Balanced panel
- Cluster-robust standard errors
- Contemporaneous correlation
- Cross-equation hypotheses
- Deviations from individual means
- Endogeneity
- Error components model
- Fixed effects estimator
- Fixed effects model
- Hausman test
- Hausman-Taylor estimator
- Heterogeneity
- Instrumental variables
- Least squares dummy variable model
- LM test
- Panel corrected standard errors
- Pooled least squares
- Pooled model
- Random effects estimator
- Random effects model
- Seemingly unrelated regressions
- Time-invariant variables
- Time-varying variables
- Unbalanced panel

Appendices

- Consider a simple regression model for cross sectional data:

$$y_i = \beta_1 + \beta_2 x_i + e_i$$

  - The variance of b_2, in the presence of heteroskedasticity, is given by:

$$\operatorname{var}(b_2) = \operatorname{var}\left(\sum_{i=1}^{N} w_i e_i\right) = \sum_{i=1}^{N} w_i^2 \operatorname{var}(e_i) + \sum_{i=1}^{N} \sum_{j=i+1}^{N} 2 w_i w_j \operatorname{cov}(e_i, e_j)$$

$$= \sum_{i=1}^{N} w_i^2 \operatorname{var}(e_i)$$

$$= \sum_{i=1}^{N} w_i^2 \sigma_i^2$$

- Now suppose we have a panel simple regression model:

Eq. 15A.1

$$y_{it} = \beta_1 + \beta_2 x_{it} + e_{it}$$

with the assumptions:

$$\operatorname{cov}\left(e_{it}, e_{is}\right) = \psi_{ts} \text{ and } \operatorname{cov}\left(e_{it}, e_{js}\right) = 0 \text{ for } i \neq j$$

- The pooled least squares estimator for β_2 is:

Eq. 15A.2

$$b_2 = \beta_2 + \sum_{i=1}^{N}\sum_{t=1}^{T} w_{it} e_{it}$$

where

$$w_{it} = \frac{x_{it} - \bar{\bar{x}}}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(x_{it} - \bar{\bar{x}}\right)^2}$$

with

$$\bar{\bar{x}} = \sum_{i=1}^{N}\sum_{t=1}^{T} x_{it} \Big/ NT$$

- The variance of the pooled least squares estimator b_2 is given by:

Eq. 15A.3

$$\text{var}\left(b_2\right) = \text{var}\left(\sum_{i=1}^{N}\sum_{t=1}^{T} w_{it}e_{it}\right) = \text{var}\left(\sum_{i=1}^{N} g_i\right)$$

with

$$g_i = \sum_{t=1}^{T} w_{it}e_{it}$$

- We can now write:

Eq. 15A.4

$$\begin{aligned}\operatorname{var}(b_2) &= \operatorname{var}\left(\sum_{i=1}^{N} g_i\right) \\ &= \sum_{i=1}^{N} \operatorname{var}(g_i) + \sum_{i=1}^{N} \sum_{j=i+1}^{N} 2\operatorname{cov}(g_i, g_j) \\ &= \sum_{i=1}^{N} \operatorname{var}(g_i)\end{aligned}$$

- To find var(g_i), suppose for the moment that T = 2, then:

$$\operatorname{var}\left(g_i\right)=\operatorname{var}\left(\sum_{t=1}^{2} w_{it} e_{it}\right)$$

$$=w_{i1}^2 \operatorname{var}\left(e_{i1}\right)+w_{i2}^2 \operatorname{var}\left(e_{i2}\right)+2 w_{i1} w_{i2} \operatorname{cov}\left(e_{i1}, e_{i2}\right)$$

$$=w_{i1}^2 \psi_{11}+w_{i2}^2 \psi_{22}+2 w_{i1} w_{i2} \psi_{12}$$

$$=\sum_{t=1}^{2} \sum_{s=1}^{2} w_{it} w_{is} \psi_{ts}$$

- For $T > 2$, $\text{var}(g_i) = \sum_{t=1}^{T}\sum_{s=1}^{T} w_{it} w_{is} \psi_{ts}$
  - Substituting:

Eq. 15A.5

$$\text{var}(b_2) = \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} w_{it} w_{is} \psi_{ts}$$

$$= \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\left(x_{it} - \bar{\bar{x}}\right)\left(x_{is} - \bar{\bar{x}}\right)\psi_{ts}}{\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(x_{it} - \bar{\bar{x}}\right)^2\right)^2}$$

- A cluster-robust standard error for b_2 is given by the square root of:

Eq. 15A.6

$$\widehat{\operatorname{var}}(b_2)=\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}\left(x_{it}-\bar{\bar{x}}\right)\left(x_{is}-\bar{\bar{x}}\right)\hat{e}_{it}\hat{e}_{is}}{\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(x_{it}-\bar{\bar{x}}\right)^2\right)^2}$$

- The random effects model is:

Eq. 15B.1

$$y_{it} = \overline{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + (u_i + e_{it})$$

- We transform the panel data regression into "deviation about the individual mean" form:

Eq. 15B.2

$$y_{it} - \overline{y}_i = \beta_2 \left(x_{2it} - \overline{x}_{2i} \right) + \beta_3 \left(x_{3it} - \overline{x}_{3i} \right) + \left(e_{it} - \overline{e}_i \right)$$

- A consistent estimator of σ^2_e is:

Eq. 15B.3

$$\hat{\sigma}_e^2 = \frac{SSE_{DV}}{NT - N - K_{slopes}}$$

- The estimator of σ^2_u requires a bit more work
  - Write:

Eq. 15B.4

$$\bar{y}_i = \bar{\beta}_1 + \beta_2 \bar{x}_{2i} + \beta_3 \bar{x}_{3i} + u_i + \bar{e}_i \quad i = 1, \ldots, N$$

  - This estimator is called the **between estimator**
    - It uses variation between individuals as a basis for estimating the regression parameters
    - This estimator is unbiased and consistent, but not minimum variance under the error assumptions of the random effects model

- The error term has homoskedastic variance:

Eq. 15B.5

$$\operatorname{var}\left(u_i+\bar{e}_i\right)=\operatorname{var}\left(u_i\right)+\operatorname{var}\left(\bar{e}_i\right)=\operatorname{var}\left(u_i\right)+\operatorname{var}\left(\sum_{t=1}^{T} e_{it} \Big/ T\right)$$

$$=\sigma_u^2+\frac{1}{T^2}\operatorname{var}\left(\sum_{t=1}^{T} e_{it}\right)=\sigma_u^2+\frac{T\sigma_e^2}{T^2}$$

$$=\sigma_u^2+\frac{\sigma_e^2}{T}$$

- An estimate of the variance is:

Eq. 15B.6

$$\overset{\square}{\sigma}_u^2 + \frac{\sigma_e^2}{T} = \frac{SSE_{BE}}{N - K_{BE}}$$

  - Therefore:

Eq. 15B.7

$$\hat{\sigma}_u^2 = \overset{\square}{\sigma}_u^2 + \frac{\sigma_e^2}{T} - \frac{\hat{\sigma}_e^2}{T} = \frac{SSE_{BE}}{N - K_{BE}} - \frac{SSE_{DV}}{T\left(NT - N - K_{slopes}\right)}$$